# 4.3 Completed Notes

4.3: Greatest Common Divisor and Least Common Multiple

Definition: The greatest common divisor (GCD) of two natural numbers *a* and *b* is the greatest natural number that divides both *a* and *b*.

Intersection of Sets Method: We write the set of all divisors of each number and then intersect these sets to find the common divisors. The largest element of the intersection is the greatest common divisor.

Example: Find the GCD of 12 and 18.

$D_{12}=\{1,2,3,4,6,12\}$

$D_{18}=\{1,2,3,6,9,18\}$

$\boxed{GCD(12,18)=6}$

$D_{12}\cap D_{18}=\{1,2,3,6\}$

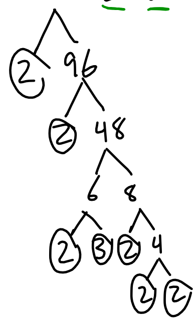
Prime Factorization Method: We find the prime factorization of both numbers. The GCD is the product of the **common** primes, raised to the **lowest** power that shows up in either prime factorization.

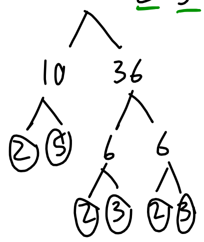
Note: If there are no common primes, then the only common factor is 1, so the GCD of these two numbers is 1. In this case, we say that the numbers are relatively prime.

Example: Find the GCD of 192 and 360.

$192=2^6\cdot 3^1$

$360=2^3\cdot 3^2\cdot 5$





$GCD(192,360)=2^3\cdot 3^1=\boxed{24}$

Example: Use the Intersection of Sets and Prime Factorization methods to find the GCD of 56 and 84.

$D_{56}=\{1,2,4,7,8,14,28,56\}$

$D_{84}=\{1,2,3,4,6,7,12,14,21,28,42,84\}$

$D_{56}\cap D_{84}=\{1,2,4,7,14,28\}$

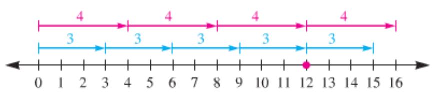
$GCD(56,84)=\boxed{28}$

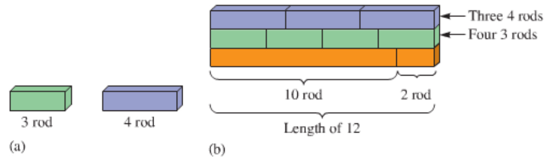
$56=2^3\cdot 7^1$ $\qquad 84=2^2\cdot 3^1\cdot 7^1$

$GCD(56,84)=2^2\cdot 7^1=\boxed{28}$

Definition: The least common multiple (LCM) of two natural numbers *a* and *b* is the least natural number that is both a multiple of *a* and a multiple of *b*.

**Not Tested:** (Number-Line Method/Colored Rods Method) Find the LCM of 3 and 4.




Intersection of Sets Method: We write the set of all multiples of each number and then intersect these sets to find the common multiples. The smallest element of the intersection is the least common multiple.

Example: Find the LCM of 6 and 8.

$M_6=\{6,12,18,24,30,36,42,48,54,60,\ldots\}$

$M_8=\{8,16,24,32,40,48,56,64,\ldots\}$

$M_6\cap M_8=\{24,48,72,96,\ldots\}$

$LCM(6,8)=24$

Prime Factorization Method: We find the prime factorization of both numbers. The LCM is the product of all of the primes in **either** number, raised to the **greatest** power that shows up in either prime factorization.

Example: Find the LCM of 840 and 792.

$840=2^3\cdot 3\cdot 5^1\cdot 7^1$

$792=2^3\cdot 3^2\cdot 11^1$

$LCM(840,792)=2^3\cdot 3^2\cdot 5^1\cdot 7^1\cdot 11^1$

$=27720$

# 4.3 Completed Notes

Example: Use the Intersection of Sets and Prime Factorization methods to find the LCM of 12 and 14.

$M_{12}=\{12,24,36,48,60,72,84,96,108,120,\ldots\}$

$M_{14}=\{14,28,42,56,70,84,98,112,126,\ldots\}$

$LCM(12,14)=\boxed{84}$

$12=2^{2}\cdot 3^{1}$

$14=2\cdot 7^{1}$

$LCM(12,14)=2^{2}\cdot 3^{1}\cdot 7=\boxed{84}$

Theorem: For any two natural numbers *a* and *b*, GCD(*a*, *b*) $\times$ LCM(*a*, *b*) = $a \times b$.

Proof: See handout on Blackboard. It will appear shortly after class.

Example of why this works: Consider the numbers 8 and 12.

$8 = 2^3 = 2^3 \times 3^0$

$12 = 2^2 \times 3^1$

By considering a $3^0$, we can take all primes and look at the smallest powers for the GCD and the largest powers for the LCM.

$GCD(8, 12) = 2^2 \times 3^0$

$LCM(8, 12) = 2^3 \times 3^1$

So, $GCD(8,12) \times LCM(8,12) = 2^5 \times 3^1 = (2^3 \times 3^0) \times (2^2 \times 3^1) = 8 \times 12$.
Notice that for each prime, the GCD picks one power and the LCM picks the other power.

$4411_{\text{five}} \div 123_{\text{five}}$

$$\begin{array}{r} 30 \\ 123\,\overline{)\,4\,4\,1\,1} \\ -\,4\,2\,4 \\ \hline 1\,2\,1 \\ -\,0 \\ \hline 1\,2\,1 \end{array}$$

$\boxed{30_{\text{five}}\,R\,121_{\text{five}}}$

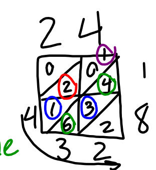
$$\begin{array}{r} 11 \\ 123 \\ \times\ \ 2 \\ \hline 301 \end{array} \qquad \begin{array}{r} 11 \\ 123 \\ \times\ \ 3 \\ \hline 424 \end{array} \qquad \begin{array}{r} 22 \\ 123 \\ \times\ \ 4 \\ \hline 1102 \end{array}$$

$4330_{\text{five}} - 1432_{\text{five}}$

$$\begin{array}{r} 4330 \\ -1432 \\ \hline \end{array} \xrightarrow{+3} \begin{array}{r} 4333 \\ -1440 \\ \hline \end{array} \xrightarrow{+10} \begin{array}{r} 4343 \\ -2000 \\ \hline \boxed{2343_{\text{five}}} \end{array}$$

$$\begin{array}{r} 1 \\ 1432 \\ +\ \ 3 \\ \hline 1440 \end{array} \qquad \begin{array}{r} 11 \\ 1440 \\ +\ 10 \\ \hline 2000 \end{array} \qquad \begin{array}{r} 3\ \ 12\ 12\ 10 \\ \cancel{4}\cancel{3}\cancel{3}\cancel{0} \\ -1432 \\ \hline \boxed{2343_{\text{five}}} \end{array}$$

$24 \times 18$



$\boxed{432}$

*see board

*these hold the place values for $20\times8$ and $4\times10$.

*holds the place value for $20\times10$.

*this is a regroup from the tens diagonal